DERIVED DEMAND AT DISAGGREGATES: IN MANUFACTURING SECTOR OF PUNJAB

Sohail Zafar [a, ψ] *and Eatzaz Ahmed* [b]

[a] *International Islamic University, Islamabad, Pakistan*
[b] *Quaid-i-Azam University, Islamabad, Pakistan*

Abstract
Factor demand is essentially a derived demand because a profit maximizing firm's demand for factor is derived from demand of final product it produce. Since the firm's optimal choice of a bundle of factors depend on the cost minimization tactic for a given level of output. The study investigates elasticities of substitution in large scale manufacturing sector of Punjab. The study employs translog cost function to examine input substitution with capital labor and raw material as inputs, over the period of 1969-70 to 1990-91. The translog cost function along with share equations are estimated for manufacturing sector at disaggregates. It turns out that labor is most responsive factor to change in factor price. The study suggests that long run economic growth could be achieved through input adjustment policies.

Keynotes: Derived demand; Elasticities of substitution; Translog cost function; Disaggregates.

JEL Classification Codes: B21; D21; D24; D57.

1. Introduction

The appropriate factor proportion for the manufacturing sectors of developing countries have engrossed awareness of researchers for more then three decades. Evidence suggests that greater labor intensity in LDC manufacturing is feasible and would be efficient. How ever in many countries lack of appropriate incentives and effective competition in factor markets has produced input use inefficient. Factor demand is essentially a derived demand because a profit maximizing firm's demand for factor is derived from demand of final product it produce. Since the firm's optimal choice of a bundle of factors depend on the cost minimization tactic for a given level of output. The derived demand for factors depends on the level of output the substitution possibilities among factors in production allowed by production technology at relative prices of factors.

Knowledge of the substitution possibilities among factors in production is predominantly important if one is interested in deriving implication for policy which sway the relative price of factors. More generally of substitution possibilities among factors in production are limited then adjustment by industry to higher factor prices will be somewhat difficult and significant where as change in underlying technological structure may be required.

Pakistan is one of the developing countries where labor is abundant but capital and raw material are scarce. Our findings greatly undermine the validity of estimates of elasticities of demand and substitution, which are based on classical supposition that factor markets are perfectly competitive. Thus for example, the validity of elasticity estimates in some of the well known studies like Kazi *et al* (1976) Kemal (1981) Battese and Malik (1987, 1988) Malik *et al* (1989) Mahmood (1989, 1992) Zahid *et al* (1992) Battese *et al* (1993) and Khan and Rafiq (1993) is undermined.

Studies based on production function can be bifurcated in either the studies based on traditional production function or cost function. Essentially both the forms include two factor inputs either capital or labor. The earliest known work on factor substitution in Pakistan manufacturing industries is by Kazi *et al* (1976). The study utilized constant elasticity of substitution production function later on Battese and Malik (1987,

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ψ Corresponding author. Sohail Zafar. Department of Business Administration, Faculty of Management Sciences, International Islamic University, Sector H-10, Islamabad Pakistan. Email: s.zafar@usa.com

1988) Zahid *et al* (1992) and Battese *et al* (1993) also used CES production function. Kazmi (1981) Kemal (1981) and Battese and Malik (1988) estimated both techniques either CES and VES production function to explore elasticities of substitution between labor and capital at different level. The studies heavily depend on the assumption that factor markets are perfectly competitive in Pakistan, and was limited to only two inputs. Table 1 summarizes the findings of all the major studies based on CES or VES production function.

To over come some of the inherent weaknesses in CD CES and VES. We use translog cost function, which does not work with such a restrictive structure. The estimated parameters of translog cost function can be used to derive the elasticities of substitution and price elasticities of demand for factors of production.
Table 1 shows that all the studies estimate a positive value of elasticity of substitution between labor and capital. This is an obvious limitation of CES or VES technology. Overall with CES production function elasticity of substitution revolves near unity under constant returns to scale and less than unity under variable return to scale. In all but one study the elasticity ranges from 0.67 to 0.77 with VES production function under constant returns to scale Kazmi (1981) estimates it quite large 6.0 under variable returns to scale for small scale industries. This could be due to price distortions because this affects the allocation of resources between different activities, while according to Kemal (1981) it was 0.88 for large scale manufacturings. Zahid *et al*. (1992) worked out returns to scale and observed that most of the industries included in the sample experienced diminishing return to scale. On technical change the study found that consumer goods industry appears to have a very little technical change.

Table 1: Substitution elasticities between labor and capital estimated for Pakistan based on restricted form of production functions.[1]

Study	Period	Data Source, Base and Type	Functional Form	Substitution Elasticity Between Labor and Capital
Kazi (1976)	1954 to 1969-70[1]	CMI, Pakistan level, time series	CES (CRS) CES (VRS)	1.17 0.72
Battese and Malik (1987)	1969-70 to 1976-77[5]	CMI[2], Punjab & Sindh[3], time series[4]	CES (CRS) CES (VRS)	1.31 1.02
Zahid, Akbar and Jaffry (1992)	1960 to 1986	CMI, Pakistan Level (selected consumer Good industries except drugs and Pharmaceuticals)	CES (VRS)	$\prec 1$
Battese, Malik and Sultana (1993)	1969-70 to 1986-87[6]	CMI, Pakistan large scale food processing industries time series data	CES (CRS)	1.31
Kazmi (1981)	1975-76	Punjab small scale Industry Pakistan level (small scale, cross sectional data)	CES (CRS) CES (VRS) VES (CRS) VES (VRS)	0.74 0.33 0.77 6.00
Kemal (1981)	1959-60 to 1969-70	CMI, Pakistan level cross sectional data[7]	CES (CRS) CES (VRS) VES (VRS) VES (VRS)	0.58 0.56 0.67 0.88
Battese and Malik (1988)	1980-81	Survey of large scale selected industries, firm level cross sectional data	CES (CRS) CES (VRS) VES (CRS)	0.82 1.09 0.79

Notes: 1.With the exception of 1956, 1960-61, 1961-62, 1967-68. 2.Data on small scale and household firms was taken from a sample survey of firms which is not discussed. 3.Elasticities of few selected industries were also estimated but

[1] Source: M. Idrees MPhil Thesis QAU Islamabad

not discussed here. 4.Analyzed on cross sectional data not discussed here. 5.With the exception of 1971-72 to 1974-75. 6.Except for 1971-72 to 1974-75. 7.For some selected industries the elasticities are also computed not detailed here.

2. The model

To estimate underlying technology one can use either production or associated cost function. The choice between them is a matter of statistical convenience Zafar and Ahmed (2005). We use cost function, which is more suitable than production function. The reason for using cost function is that it yields direct estimates of Allen-Uzawa elasticities of substitution. These parameters are key to describe pattern and degree of substitutability and complementarities among factors of production assuming that classical assumption of perfect competition in input market holds that is there is no allocative inefficiencies. The translog cost function is obtained as second order Taylor series approximation. The function is highly flexible it allows scale economies and input proportions to vary with level of output. Furthermore elasticities of substitution between inputs are also variable with respect to output level and input prices. For translog cost function we denote factor prices by p their respective quantities by x and level of output by y while total cost is denoted by c which is given by $c = \sum p_i x_i$ translog cost function can be written as follows

$$\ln c = \alpha_o + \sum_{i=1}^{3} \alpha_i \ln p_i + 1/2 \sum_{i=1}^{3}\sum_{j=1}^{3} \beta_{ij} \ln p_i \ln p_j + \alpha_y \ln y + 1/2\beta_y (\ln y)^2 + \sum_{i=1}^{3} \beta_{iy} \ln p_i \ln y$$

Symmetry condition requires that $\beta_{ij} = \beta_{ji} \ \forall i \neq j$. Monotonicity holds $\beta_{yi} = 0 \forall i$ when all prices increase cost also increase, while Homogeneity of the cost function require the following restriction $\sum_{i=1}^{3} \alpha_i = 1$, $\sum_{i=1}^{3} \beta_{ij} = \sum_{i=1}^{3} \beta_{ji} = 0$, $\sum_{i=1}^{3} \beta_{iy} = 0$. The Hicks own and cross price elasticities of demand for input i with respect to its market price turns out $\sigma_{ii}^{h} = \kappa_i(\kappa_i - 1) + \beta_{ii}/\kappa_i \ \forall i \neq j$, $\sigma_{ij}^{h} = (\kappa_i \kappa_j + \beta_{ij})/\kappa_i \ \ \forall i \neq j$. The Allen partial elasticities of substitution turns out

$$\sigma_{ij}^{a} = (1 + \beta_{ij}/\kappa_i \kappa_j) \qquad \text{ij = k l m} \qquad \kappa \ \forall i \neq j$$

Factor i and j are substitutes if $\sigma_{ij} \succ 0$ and complements if $\sigma_{ij} \prec 0$. Allen (1938) has shown that elasticities are analytically related to the price elasticities of demand for factors of production.

3. Data

The Census of Manufacturing Industries (CMI) is the only major source of data on different aspects of manufacturing industries in Pakistan. All the data in CMI are on aggregate level and on groups of industries. Most of the information is classified according to size of the capital stock and employment in manufacturing units. Apart from this limitation CMI data also suffer with other serious loopholes such as, under coverage of manufacturing firms, changes in definitions of some of the variables over time, gaps, and irregularity. Most of the data are taken from its sixteen most recent publication (1969-71, 1970-71, 1975-76 through 1987-88 and 1990-91). Some supplementary information is collected from Monthly Statistical Bulletin and Economic survey of Pakistan.

Value of production consist of the value of finished products and by products receipts for repairs and maintenance, value of electricity sold, receipt for work done for others, value of the sale goods purchased for resale, wastes and used goods, the net increase in the value of working capital, and the value of processed and fixed assets produced by the establishment for its own use. Valuation is made at ex-factory prices, which include indirect taxes and exclude transport cost outside the factory gate.

To estimate the unit labor cost we divide the employment cost with average daily persons engaged. Measurement of capital cost and price is a difficult job as admitted by many researchers Christen and Jorgenson (1969) Burki *et al* (1997) Zafar (2000) Burki and Mahmood (2004). The most appropriate price of capital for our purpose is the user cost of capital given by $P_k = P_{k_{ind}}(r + \delta - \pi_{k_{ind}})$. Where P_k is the user cost of capital and $P_{k_{ind}}$ is price index of capital goods r is the real rate of interest δ is the capital depreciation rate and $\pi_{k_{ind}}$ is defined as $\pi_{k_{ind}} = (P_{k_{ind}t} - P_{k_{ind}t-1}) / P_{k_{ind}t-1}$. It is rate of growth in price index of capital. Thus user cost of capital increases with an increase in price of capital real rate of interest and capital depreciation rate on the other hand the user cost decreases with appreciation in the value of capital due to increase in the rate of growth in capital price. The price index of machinery is taken from Monthly Statistical Bulletin. The rate of interest is the average schedule bank rate on long term advances for manufacturing sector.

The quantity of capital is calculated by dividing value of capital by the price index of machinery. Finally multiplying quantity of capital by user cost marks out total user cost of capital. Total cost is obtained by summing up value of capital stock total employment cost and value of raw material whereas input shares are obtained by dividing cost of each input with total costs.

We estimate parameters of translog cost function along with share equations in a system of equations. We use Iterative Zellner Efficient (IZEF) method for seemingly unrelated regression equations. Since cost shares satisfy adding up restriction it means that all shares sum equal to one. To solve the problem of singularity one of these equations is dropped Berndt and Christensen (1973). Barten (1969) showed that maximum likelihood estimates are independent of the equation omitted. We drop the share equation of raw material and recover its parameters with the help of adding up restrictions. Since IZEF estimation converges to Maximum Likelihood Estimate, which is unique it follows that IZEF estimates are invariant to the choice of equation dropped.

4. Empirical findings

We jointly estimate the system of equations for the normalized profit function and factor demands, imposing linear homogeneity and symmetry restrictions. Zellner's (1962) seemingly unrelated regressions method is used along with assumptions therein about the error terms. Because the model is nonlinear in parameters we use an iterative Zellner efficient method which here after builds on a maximum likelihood estimation procedure minimizing a distance function Mahmood-ul-Hassan (1998) estimated that t-statistic are insignificant at conventional level.

Our results show that estimated cost shares at each point of data are positive confirming monotonicity, while curvature condition also hold.[2] We have tested the homotheticiy condition for translog cost model using the χ^2 test. The calculated value of statistic is 74.99, which exceeds the critical value at 5% level of significance thus homotheticiy does not hold in the estimated model. It follows that underlying production function is not homogenous either. Parameters of simple translog cost are reported in Table 2. It can be seen that most of the parameter estimates are statistically different from zero at the conventional level of significance. The coefficients of price output interaction variables β_{iy} interpret changes in input intensities as the level of output increases.

[2] The mean shares of capital labor and raw material are 0.06, 0.09 and 0.85.

Table 2: Results of translog cost function

	Pub		Pri		Ind		Par	
Parameters	Estimates	t-statistics	Estimates	t-statistics	Estimates	t-statistics	Estimates	t-statistics
α_0	14.4821	0.061854	13.9215	0.031225	12.4846	0.055235	13.7651	0.031068
α_k	0.168109	0.050562	0.133578	0.01648	0.174459	0.013133	0.034341	0.026424
α_l	0.144334	0.025185	0.053954	0.014038	-5.59E-03	0.018189	3.21E-03	0.030614
α_m	0.687557	0.039467	0.812467	0.027437	0.831131	0.025354	0.962445	0.055031
β_{ll}	-0.07668	0.017704	6.65E-03	0.013941	-0.02197	0.019921	-5.76E-03	0.026716
β_{kk}	0.050616	0.024755	0.045474	8.35E-03	0.068554	8.21E-03	3.98E-03	0.014819
β_{mm}	-0.025	0.028533	6.04E-03	0.029022	-0.03975	0.045293	-0.06645	0.07188
β_{lm}	0.076145	0.020335	0.016393	0.018305	0.065132	0.029775	0.038096	0.042873
β_{kl}	5.34E-04	0.012375	-0.02304	7.18E-03	-0.04317	0.010586	-0.03234	0.017426
β_{km}	-0.05115	0.019657	-0.02244	0.014011	-0.02539	0.0165	0.028357	0.031142
α_y	1.07083	0.092388	1.00155	0.072249	0.56165	0.097423	1.09141	0.035181
β_{yy}	0.020315	0.114132	0.089258	0.094464	-0.43846	0.080998	-4.12E-03	0.079687
β_{ly}	5.85E-03	0.010978	-0.02226	7.01E-03	-0.02008	0.019939	-0.02588	0.017881
β_{ky}	0.012775	0.01266	8.67E-03	5.53E-03	-0.01302	0.011188	8.20E-03	0.012897
β_{my}	-0.01863	0.014708	0.013592	0.01113	0.033106	0.028843	0.017683	0.029868

Notes: *: Significant at 5% level of significance. **: Significant at 10% level of significance.

Table 3: Elasticities of substitution

Allen elasticities of substitution

	Pub		Pri		Ind		Par	
Parameter	Estimates	t-statistics	Estimates	t-statistics	Estimates	t-statistics	Estimates	t-statistics
σ^a_{kl}	0.75899*	5.31665	16.1128*	16.8904	11.5547*	51.0077	129.385*	605.954
σ^a_{km}	-8.76E-03**	1.63311	2.28594	0.843558	1.10664	0.100171	-5.32182*	38.7233
σ^a_{lm}	-0.76913*	3.63264	1.21378	0.056887	3.71707*	12.7869	1.74966	1.24137

Hicks elasticities of substitution

	Pub		Pri		Ind		Par	
Parameter	Estimates	t-statistics	Estimates	t-statistics	Estimates	t-statistics	Estimates	t-statistics
σ^h_{kk}	0.041618	1.25511	-3.46033*	2.08338	-1.54169	0.02331	-1.84904*	4.4449
σ^h_{ll}	0.7244*	3.48534	-0.83695	5.46E-04	-2.06208*	5.46508	-1.05436	0.41019
σ^h_{mm}	-0.03291	0.127948	-0.05678	0.101401	0.147594	0.23848	-0.11856	0.235407
σ^h_{lk}	0.038785	0.237317	-0.30008	0.196019	-2.32867*	10.0721	-0.61008	1.25779
σ^h_{kl}	-0.03292	0.237622	1.31895	1.26572	0.234485	0.128583	6.91166*	39.2058
σ^h_{lm}	-0.76319*	3.6933	1.13703	0.169291	4.39075*	14.284	1.66445	0.807791
σ^h_{ml}	0.033362	0.09342	0.099357	0.084878	0.075432	0.11447	0.093466	0.171642
σ^h_{km}	-8.70E-03**	1.62059	2.14139	0.700502	1.30721	0.145681	-5.06262*	37.6155
σ^h_{mk}	-4.48E-04	0.082868	-0.04257	0.029925	-0.22303	0.027912	0.025094	0.050574

Notes: *: Significant at 5% level of significance. **: Significant at 10% level of significance.

In individual ownership estimated value of β_{ly} is –0.02 implying that intensity of labor is lower at the higher levels of output β_{ky} turns out -0.01 and β_{my} is 0.03 i.e. intensity of raw material is higher at higher level of output. In any case this pattern of factor intensities confirm our result that underlying production structure is non-homothetic.

Hicks cross price elasticities of factor demand are in agreement with the results of Allen partial elasticities of substitution. As one should expect, the magnitude of Hicks elasticities are smaller than corresponding Allen elasticities. This is a natural result as Hicksian cross price elasticity of factor demand is obtained by multiplying the Allen partial elasticity of substitution with average input share that is a positive fraction. Thus effect of change in price of one input on demand for another input is high if cost share of first input is large. This explains for example, why elasticity of capital with respect to price of raw material is larger than elasticities of raw material with respect to capital.

The Allen and Hicks price elasticities of demand based on estimates of translog cost share equations are calculated for each input pair and are shown in table 3. The Allen elasticity of substitution for public limited firms $\sigma^{a}_{km} \prec 0, \sigma^{a}_{lm} \prec 0$ turn out less then zero and cross price elasticities between capital and labor turn out $\sigma^{a}_{kl} \prec 1$.

The Hicks own price elasticities are of correct sign showing that with an increase in price of an input utilization decreases except for raw material in individual ownership, capital and labor in public limited firms. Raw material and labor are substitutes $\sigma^{h}_{ml} \prec 1$ this means that when wage relative to price of raw material increase firms will increase raw material intensity relative to labor. This result is quite consistent with observed factor prices and factor intensities within our sample. Over the years unit labor cost has risen faster than users cost of raw material and as a result raw material utilization has increased substantially. Measure of Hicks elasticity of capital with respect to labor is $\sigma^{a}_{kl} \prec 0$ for public limited firms it turns out a complementary relationship but the value is statistically insignificant and is in no harmony with factor prices and factor intensities within the sample.

5. Conclusion

This study has been an attempt to investigate the elasticities of substitution in large scale manufacturing's of Punjab. We estimated globally known translog cost function to explain the substitutability of different inputs. It shows that capital and labor are complement in use, while both of these inputs are substitutable with raw material. However complementarities or substitutability relationships are weak.

The Hicks own price elasticities are of correct sign. It shows an increase in price of an input utilization decreases further raw material and labor are substitutes. This means that when wage relative to price of raw material increases firms will increase raw material intensity relative to labor. This result is quite consistent with observed factor prices and factor intensities within our sample. Further comparative analysis is useful to observe how estimates of substitution elasticities are affected due to the conventional assumption that firms at disaggregates are able to minimize cost in the light of observed input prices.

For planners firms responsiveness to firm input-output prices require the government to ensure that prices announced for firm commodities preserve firm's incentive for higher production. Firm output prices need to increase overtime at least consistent with in input prices otherwise use of technological innovations like capital and employment of labor are likely to be adversely affected. Since increased application of capital depends on ample supply the government needs to ensure the availability of adequate supply which will create additional, on firm employment. To increase labor employment government should remove imperfections in wage formation.

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