OPTIMAL FINANCIAL CONTRACTS IN LEVERAGED BUY OUT: A DOUBLE-SIDED MORAL HAZARD MODEL

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Abstract
We consider a double moral hazard model with three agents: the entrepreneur, the LBO fund and the bank. The entrepreneur and the LBO fund have to exert efforts in order to improve the productivity of their project; efforts are not observable. We show that the bank's payments decrease with the outcome of the project. When the project is not very risky, the entrepreneur and the LBO fund exert first best efforts and they get equal shares of the project's outcome. When it is highly risky, debt gives high powered incentives to the two agents to provide efforts but it still not sufficient to induce them to provide the first best efforts. However, these efforts are more efficient than those that could be provided if the entrepreneur asks the LBO fund for advice and money. Moreover, when the entrepreneur asks for advice from a consultant and for money from a bank, they get equal shares whether the project is very risky or not. When the project is lowly risky, the identity of the advisor (consultant/ LBO fund) is irrelevant. When it is highly risky, the optimal financial structure of capital depends on the impact of their efforts on the performance of the project.

1. Introduction

Leveraged Buy Out acquisition commonly known as LBO[1] accounts for a significant part of the corporate finance and plays a major role in structuring mergers, acquisitions and transmissions. There are three main facts about LBO finance: first, the LBO fund (hereafter he) is an active investor and he is well connected with many industries: he is engaged in the day to day operations of the firm, he helps to recruit key personnel, he negotiates with suppliers, bank (hereafter he) and other financial partners and he advises the entrepreneur (hereafter she) on all the strategic decisions. Second, the level of debt is significantly high. Moreover, there are many kinds of debt, such as the mezzanine debt, the subordinate debt and the convertible debt, each debt has a specific level of risk. Finally, the use of convertible securities becomes prevalent in LBO finance. It is surprising because these securities are very rarely issued by banks or passive outside equity holders who finance more established and less risky companies. This paper provides a theory of LBO financing based on a contractual approach. The theoretical model that we present deals with the two first facts. The model describes the relationship between the LBO fund and the entrepreneur engaged in the acquisition of a firm. The question raised in this paper is the following: why does the entrepreneur prefer asking for financing and for advice from the LBO fund and the bank, while the LBO fund alone is able to advise her and to finance the project?

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[1] LBO is an acquisition of a company, called the Op Co, using mostly debt and a small amount of equity. The debt is secured by the Op Co assets. The acquiring company, called the holding company or the New Co, uses these assets as collateral for the debt in hopes that the future cash-flows will cover the debt payments.

The LBO fund is usually not wealth-constrained, like the private equity funds: these funds participate in the formation of large companies and they issue high level of equity. In Venture capital[2], the venture capitalist fund (hereafter VC fund) is usually the only financier in the project and he is also providing advice. To answer the question, we consider three agents: the entrepreneur, the LBO fund and the bank. The entrepreneur is the manager of the Op Co and she wants to acquire it. The entrepreneur is wealth-constrained this is why she asks first for advice and for money from the LBO fund. These two partners sign a first contract: the holding contract and they establish the holding company (hereafter the holding). They can also ask for additional funding from the bank and then they sign a second contract: the debt contract. The entrepreneur and the LBO fund have to exert non observable efforts which induce a double-sided moral hazard problem.

We study the impacts of financial capital structure on the efforts when there is a double-sided moral hazard problem. To our knowledge, there is no such study applied on LBO. However, a number of papers study this question on the venture capital. Bergemann and Hege (1998) present a dynamic agency model with learning and moral hazard problems. They show that short-term refinancing[3] is never optimal but long-term contract allowing for intertemporal risk-sharing such as the stage financing[4] is optimal. This financial regime constrains the entrepreneur to provide optimal effort so that she could obtain funding for his project. If not, the VC fund will not invest additional capital and will abandon the project. This may be interpreted as a bad signal concerning the quality in the market. Cornelli and Yosha (2003) show that the stage financing may induce a *window dressing* problem: the entrepreneur is tempted to announce a good short-term performance. Her aim is to reduce the probability that the project will be abandoned or liquidated. They conclude that the use of the convertible debt solves the "window dressing" problem: when the project looks too profitable, the VC fund will convert his debt into equity which reduces the entrepreneur's profit. Schmidt (1999, 2003) does not study the stage financing regime but he focuses on the incentive properties of the convertible securities. He proposes a model where the profit that can be generated by the entrepreneur and the VC fund depends on three factors: the quality of the project and/or the abilities of the entrepreneur (the state of the world), their efforts, and further financial investment of the VC fund. The state of the world has an impact on the results of the project. It is unknown to both parties at date 0 and can be observed only after the initial investment has been sunk. Schmidt shows that there is no debt-equity contract that induces both parties to invest efficiently. He joins Cornelli and Yosha and he deduces that convertible securities constraint both agents to exert optimal efforts.

My model is closely related to those of Casamatta (2003) and Repullo and Suarez (2004). They present a double-sided moral hazard model with a pure financier. Casamatta shows that when efforts are observable, whether the advisor is a consultant or a VC fund is irrelevant. There are many ways to implement the first best. This is no longer true when efforts are non observable. The consultant effort is less efficient than her proper effort. Thus, she prefers asking for the VC fund's advice. In order to solve the double moral hazard problem, all agents must participate financially in the project. When the project is not very risky, the presence of the pure financier induces both agents to exert first best efforts. Otherwise, they provide the second best efforts. Repullo and Suarez consider a stage financing model. The entrepreneur is wealth-constrained and she asks for advice and for money from two VC funds. One of them does not provide effort so he may be considered as a pure or passive financier. The entrepreneur and the other fund have to exert non observable efforts. They conclude that the entrepreneur must ask for advice and fund from the partner who provides both money and advice.

We consider a single-period model and that the entire cost of the project is invested when the contracts are signed. We determine endogenously the financial contribution of each agent and its share of the project return.

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[2] The venture capital is a type of the private equity capital provided by professional, outside investors to new high-potentiel-growth companies. The objective is to take the company to an IPO (*Initial Public Offering*).

[3] The entrepreneur signs a new contract with a new VC fund, but only for one period. The next period, she is looking for a new partner; the VC market is supposed to be competitive.

[4] While the commitment is to fund the entire amount, in venture capital' projects, the funding is contingent on the company attaining his short-term objectives. At each stage, the entrepreneur asks for additional capital from the VC fund.

We focus particularly on the presence of the bank and its impact on the incentives to effort. We show that the LBO fund must issue a significant part of equity. The payment of the bank decreases with the outcome of the project. We join Casamatta (2003), Jensen (1986, 1989), Jensen and Meckling (1976)... We show that the LBO debt induces the agents to exert efficient efforts and it reduces agency costs. Furthermore, the way the project's outcome is shared between the agents, the bank's payments and the provided efforts depend on the quality of project but not on the financial capital structure:

- When the project is not very risky, we generalize the Casamatta's model and we show that both agents exert the first best efforts. We are able to compare the gains of the entrepreneur and the LBO fund when the project succeeds and when it fails. We find that they must get equal shares of the outcome of the project.

- When the project is very risky, the entrepreneur and the LBO fund provide second best efforts. Double moral hazard problem induces them to make less efficient investment decisions. We conclude that the sharing rule depends on the impact of each effort on the performance of the project.

In addition, the entrepreneur and the LBO fund make less efficient efforts when the bank does not participate in the project. Moreover, when the LBO fund participates in the project without issuing equity, they get equal gains whether the project is very risky or not. If the project is lowly risky, whether the adviser is the consultant or the LBO fund is irrelevant, the entrepreneur and her advisor provide the first best efforts. When it is highly risky, the optimal financial structure depends on the impact of each effort on the performance of the project.

The model and the assumptions are presented in section 1. Section 2 solves the model and derives the properties of the financial optimal contracts. Section 3 focuses on the properties of financial contracts when the entrepreneur and the LBO fund participate in the project without the bank and when the LBO fund does not contribute financially to the project. We compare these contracts with those derived from the previous section. The last section concludes the paper. All proofs are presented in Appendix.

2. The model

We consider an entrepreneur E who wants to acquire a company where she is the manager. The cost of project is equal to K. She invests W but she is wealth-constrained. So she asks first for advice and money from the LBO fund A. She concedes a part of the company's control $1-\beta$ $(0 \le \beta \le 1)$ and he issues equity i. These two partners may ask for additional financing from the bank B. The latter may lend them an amount $I = K - (W + i)$ which corresponds to a debt D. The project is risky and generates an observable random revenue θ. It depends on the quality of the project, the entrepreneur skills, the market conditions... It can take two values: θ^u when the project succeeds and θ^d when it fails where $\theta^u > \theta^d$. When the project fails, θ^d is equal to its liquidation value. The probability of success is denoted $p(e, a)$, where e and $a \in \Re_+$ are the efforts provided respectively by the entrepreneur and the LBO funds. The entrepreneur's effort is related to technical skills. The LBO fund may exert a technical effort or a managerial effort such as the monitoring. We assume that the probability function $p(e, a)$ is increasing and concave. We add the condition $\frac{\partial^2 p(e, a)}{\partial e \partial a} > 0$ so that we ensure that efforts are complementary. Furthermore, $p(e, 0) = p(0, a) = 0$: it means that both agents must exert strictly positive efforts so that the probability of success would be strictly positive. Making efforts $p(e, a)$ is a Nash equilibrium but it means that the project fails with probability one.

The sequence of events in the model
At the date 0, E and A negotiate and sign a first contract: the holding contract. The holding and the bank B sign a second contract: the debt contract. There is a continuum of LBO fund and banks.
At the date 1, E and A make the non contractible efforts e and a. Let $u(e)$ and $v(a)$ denote respectively their cost functions. These functions are increasing, convex and satisfy $u(0) = v(0) = u'(0) = v'(0) = 0$.

The date 2 is the exit date. If the project succeeds, the bank gets the payment D and the two agents share the residual amount; they get respectively $\beta(\theta^u - D)$ and $(1-\beta)(\theta^u - D)$. Otherwise, the bank perceives the collateral $H\left(H \leq \theta^d\right)$. If $H = \theta^d$, the entrepreneur and the LBO fund get zero payoffs. If $H < \theta^d$, they obtain respectively $\beta(\theta^d - H)$ and $(1-\beta)(\theta^d - H)$.

All agents are risk neutral and the risk-free rate is normalized to 0. The sequence of events is summarized in the following figure:

Figure 1 - The sequence of events in the model.
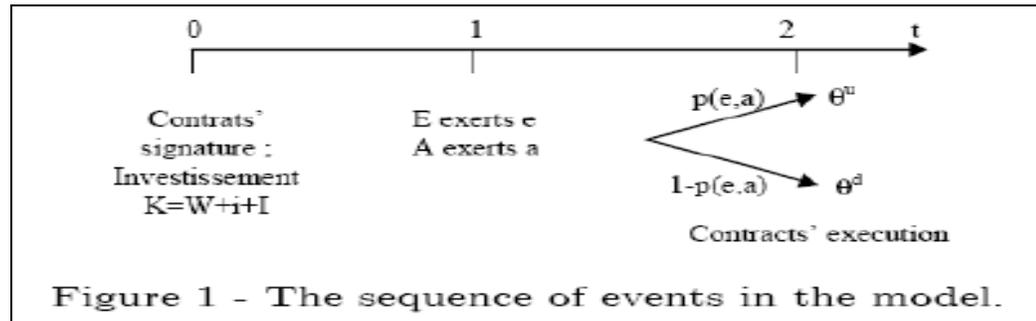
Financial contracts

1. The holding contract: it determines the amount of equity that must be issued by the LBO fund and his share of benefits. He participates in the project if his revenue is positive. This constraint is written:

$$EU^A = (1-\beta)[p(e,a)(\theta^u - D) + (1 - p(e,a))(\theta^d - H] - v(a) - i \geq 0 \qquad (PC_A)$$

2. where (PC_A) is the participation constraint of the LBO funds. The agent E asks for money and for advice from many LBO funds. He deals with the one who makes the best offer (i, β). This offer gives him a zero payoff; (PC_A) is satisfied when $EU^A = 0$.

3. The debt contract: the bank lends I at the date 0. At the date 3, he is paid D when the project's result is success, otherwise he gets H. The bank is willing to lend only if he will recoup what he would get if he makes a risk-free investment. It means that:

$$EU^B = p(e,a)D + (1 - p(e,a))H - I \geq 0 \qquad (PC_B)$$

where (PC_B) is the participation constraint of the bank, it is satisfied when $EU^B = 0$.

The model without double moral hazard

The social value of the project $V(e,a)$ is given by:

$$V(e,a) = p(e,a)\Delta\theta + \theta^d - u(e) - u(a) - K$$

where $\Delta\theta = \theta^u - \theta^d$. The first best efforts e^{FB} and a^{FB} are deduced from the first order conditions of $V(e,a)$. They are given respectively by:

$$\left.\frac{\partial V(e,a)}{\partial e}\right|_{e^{FB}, a^{FB}} = p_e(e^{FB}, a^{FB})\Delta\theta - u'(e^{FB}) = 0 \qquad (1)$$

$$\left.\frac{\partial V(e,a)}{\partial e}\right|_{e^{FB}, a^{FB}} = p_a(e^{FB}, a^{FB})\Delta\theta - u'(a^{FB}) = 0 \qquad (2)$$

The equations (1) and (2) can be written:

$$\Delta\theta = \frac{u'(e^{FB})}{p_e(e^{FB}, a^{FB})} = \frac{u'(a^{FB})}{p_a(e^{FB}, a^{FB})} \tag{3}$$

The ratios of the marginal cost to the marginal probability of efforts are equal to the difference between the revenue in case of success and the revenue in case of failure. The probability of success when the agents provide first best effort is denoted $p^{FB} = p(e^{FB}, a^{FB})$. Then, the optimal social value of the project is given by:

$$V^{FB} = p^{FB}\Delta\theta + \theta^d - u(e^{FB}) - v(a^{FB}) - K$$

V^{FB} is assumed to be strictly positive. The first best solution can be implemented in a number of ways: the entrepreneur and the LBO fund may ask money from the bank or the LBO fund may invest $i = K - W$.

3. The optimal financial contracts with double moral hazard problem

We suppose now that efforts are non observable. The entrepreneur (respectively the LBO fund) exerts the level of effort e (respectively a) that maximizes her (respectively his) expected profit given its cost and the contracts established: hence, each effort is increasing with the other one. The reaction functions of E and A are given by their incentive constraints. They are respectively written:

$$e(a) \in \frac{\arg\max}{a \in R_+} \beta[p(e,a)(\theta^u - D) + (1 - p(e,a))(\theta^d - H)] - u(e) - W \tag{IC_E}$$

$$a(e) \in \frac{\arg\max}{a \in R_+} (1 - \beta)[p(e,a)(\theta^u - D) + (1 - p(e,a))(\theta^d - H)] - v(a) - i \tag{IC_A}$$

The effort of the entrepreneur (respectively the LBO fund) is an increasing function of the effort of the LBO fund (respectively of the entrepreneur): the efforts are strategic complements.

$$\beta(\Delta\theta - D + H) = \frac{u'(e)}{p_e(e,a)} \tag{4}$$

$$(1 - \beta)(\Delta\theta - D + H) = \frac{u'(a)}{p_a(e,a)} \tag{5}$$

Competition among the LBO funds and the banks induces A and B to propose respectively holding and debt contracts that maximize the expected profit of the entrepreneur. Consequently, the entrepreneur's program is written :

$$(B*, i*, I*, D*, H*) \in \frac{\arg\max}{\beta, i, I, D, H} EU^E = \beta[p(e,a)(\Delta\theta - D + H) + \theta^d - H] - u(e) - K + i + I$$

$$s.t. \qquad (PC_A), (PC_B), (IC_A) and (IC_B)$$

The participation constraints (PC_A) and (PC_B) are binding, so the amounts of equity and debt issued respectively by the LBO fund and the bank are given by:

$$i = (1 - \beta)[p(e,a)(\Delta\theta - D + H) + \theta^d - H] - v(a) \tag{7}$$

$$I = p(e,a)(D - H) + H \tag{8}$$

We replace (7) and (8) in the objective function. The optimal financial contracts induce the entrepreneur to maximize the social value of the project under the incentive constraints (IC_E) and (IC_A). Hence the entrepreneur's program is:

$$\frac{\max V(e,a)}{\beta, D, H} = p(e,a)\Delta\theta + \theta^d - \upsilon(a) - K$$

s.t. (4), (5), with the conditions (6).

The social value of the project is optimal if the agents exert first best efforts, in other words, the efforts e^{FB} and a^{FB} must satisfy the incentive constraints (4) and (5). Given the equations (3), (4) and (5), we deduce:

$$\Delta\theta = \beta(\Delta\theta - D + H) = (1-\beta)(\Delta\theta - D + H) \tag{9}$$

It means that the benefit share of the entrepreneur is equal to $\beta = \dfrac{1}{2}$. The revenues of the project are fixed, if the optimal contract gives powerful incentives to one agent, it will reduce the incentives of the other agent. The optimal financial contracts must boost the two agents' incentives to provide first best efforts; they must get equal profit shares. We notice that this sharing rule depends neither on the capital structure, nor on the efforts' efficiency. When we substitute $\beta = \dfrac{1}{2}$ in (9), we get:

$$H = \Delta\theta + D$$

This means that $\Delta\theta + D < \theta^d$ $D < 2\theta^d - \theta^u$, but $D \geq 0$, in other words, the project must be not very risky in the sense $\theta^u \leq 2\theta^d$. We conclude that the bank payments are decreasing with the outcome of the project: the collateral is larger than his payment when the project succeeds.

Preposition 1 *When the project is not very risky* $(\theta^\upsilon \leq 2\theta^d)$, *under the conditions* $\theta^d \in [l + \max$ $\{\upsilon(a^{FB}) - p^{FB}\Delta\theta, 0\}, 21 - p^{FB}\Delta\theta]$ *and* $\theta^\upsilon \in [l, 21]$, *the optimal financial contracts are given by* :

$$H^* = 2[K - W + \upsilon(a^{FB})] - \theta^d$$
$$D^* = 2[K - W + \upsilon(a^{FB})] - \theta^\upsilon$$
$$I^* = 2[K - W + \upsilon(a^{FB})] - p^{FB}\Delta\theta - \theta^d$$
$$and \qquad i^* = V^{FB} + W + \upsilon(e^{FB}) - \upsilon(a^{FB})$$
$$\beta^* = \frac{1}{2}$$

where $1 = K - W + \upsilon(a^{FB})$.

When the project is not very risky, the LBO fund issues a high amount of equity. Moreover the LBO fund and the entrepreneur exert first best efforts and they get equal shares of the benefit. Although i is larger than W, their payments are equal. They get $\theta^u + W - K - v(a^{FB})$, if the project succeeds and $\theta^d + W - K - v(a^{FB})$ when it fails. The difference between these payments is equal to the difference between the revenues of the project $\Delta\theta < \theta^d$. The intuition of the proposition (1) is the following. In order to induce the agents to provide optimal efforts, the bank payments must be decreasing with the outcome of the project. Indeed, despite the fact that $\Delta\theta$ is not very large, they are constrained to provide first best efforts in order to perceive the highest payments; the success payments. We conclude that when the project is not very risky in the sense defined in proposition (1), the presence of the bank constrains the agents to exert optimal efforts. In addition, the entrepreneur captures the whole social value of the project while the LBO fund and the bank recoup the costs of their investments. When the project is very risky $\left(\theta^u > 2\theta^d\right)$, the collateral H is equal to the liquidation value of the project θ^d.

The general model does not enable us to study the properties of these contracts when the project is highly risky, this is why we rely on the following specification of the general model. We consider that:

$$p(e,a) = e^{1-\alpha}a^{\alpha}, \ \upsilon(e) = \frac{e^2}{2\lambda} \text{ and } \upsilon(a) = \frac{a^2}{2\mu}$$

where $\alpha \in \left]0,1\right[$ measures the impact of the effort of the LBO fund on the success' probability, e and a take values on $\left[0,1\right]$ and $\lambda > \mu > 0$: the LBO effort is more costly (less efficient) than the entrepreneur effort. The first best efforts are given by:

$$e^{FB} = \phi(\alpha)\Delta\theta \text{ and } a^{FB} = [\frac{\mu\alpha}{\lambda(1-\alpha)}]^{\frac{1}{2}}\theta(\alpha)\Delta\theta$$

where $\phi(\alpha) = [\lambda(1-\alpha)]^{1-\alpha/2}[\mu\alpha]^{\frac{\alpha}{2}}$. These efforts are increasing with the difference between the revenues of the project. When the project is very risky, the first best efforts are larger than those that they might make when the project is not very risky. Furthermore, when the impact of the effort of the LBO fund on the success probability is high such that $\alpha \geq \frac{\lambda}{(\mu+\lambda)}$, his first best effort is larger than the effort provided by the entrepreneur. Otherwise, the entrepreneur provides the highest effort. We denote by $p^{FB} = \frac{\phi^2(\alpha)}{\lambda(1-\alpha)}\Delta\theta$ the probability of success of the project when agents provide the first best efforts. When efforts are non observable, they are given by the reaction functions of the entrepreneur and the LBO fund:

$$e^* = [\lambda\beta(1-\alpha)\rho(\beta)^{\frac{1}{2}}(\theta^{\upsilon} - D) \tag{10}$$

$$a^* = [\mu\alpha(1-\beta)\rho(\beta)]^{\frac{1}{2}}(\theta^{\upsilon} - D) \tag{11}$$

where $p(\beta) = [\lambda\beta(1-\alpha)]^{1-\alpha}[\mu\alpha(1-\beta)]^{\alpha}$. The efforts e^* and a^* are increasing with the success revenue and they are decreasing with the success payment of the bank D. When the revenue of success is high, the incentives to efforts are important. Indeed, the entrepreneur and the LBO fund are tempted to get the attractive payments: the success payments. These gains increase with θ^u and decrease with D. Given the competition among the LBO funds, $EU^A = 0$. The project succeeds with a probability $p^* = \rho(\beta)(\theta^u - D)$. We assume that $(\theta^u - D) < \rho^{-1}(\beta)$ to ensure that the success probability is always inferior to 1. If the financial contracts attribute all the gain to entrepreneur or to the LBO fund, the project will fail with probability equal to $1\,(\rho(0) = \rho(1) = 0)$. This enables us to conclude that: $0 < \beta < 1$.

Preposition 2 *When the project is very risky,* $(\theta^u \rangle 2\theta^d)$ *, the optimal financial contracts are given by:*

$$\hat{D} = 0$$
$$\hat{H} = \theta^d$$
$$\hat{I} = \theta^d(1 - \theta^u \rho(\hat{\beta}))$$

and

$$\hat{i} = (1 - \frac{1}{2}\alpha)(1 - \hat{\beta})\rho(\hat{\beta})(\theta^u)^2$$

$$\hat{\beta} = \begin{cases} \dfrac{\theta^d - \theta^u(1-\alpha)(2-\alpha) + \sqrt{\Delta}}{2\theta^u(2\alpha - 1)} & \text{if } \alpha \in]0,1[/\left\{\dfrac{1}{2}\right\}, \ \dfrac{1}{2} \text{ if } \alpha = \dfrac{1}{2} \\ \\ \end{cases}$$

where $\Delta = \alpha(1-\alpha)\theta^u\left[(1+\alpha)(2-\alpha)\theta^u - 6\theta^d\right] + (\theta^d)^2$

This proposition shows that the optimal sharing rule β is a function of α. When $\alpha > \dfrac{1}{2}$, the LBO fund's share of benefit is larger than the entrepreneur's share. Despite the fact that the LBO fund's effort is more costly than the entrepreneur effort, it has the most important impact on the probability of success. It is more efficient to give him powered incentives to provide effort; like for example attributing him the highest share of the project's outcome. When the impact of LBO fund's effort on the probability success is low, it is efficient to give powered incentives to the agent who provides the less costly effort. When the project is highly risky, the agents make less efficient efforts. The double moral hazard induces the agents to make less efficient efforts: the levels of first best efforts are high. We need powerful incentive mechanisms to constrain the entrepreneur and the LBO to exert the optimal efforts. The revenue should be shared between them in the case of success, and the entire revenue should be attributed to the bank when the project fails. However it is still not sufficient to induce them to spend the first best efforts.

4. Financial capital structure: debt or equity?

The question raised in this section is the following: *Is it the debt or the fact of relying on a financial partner who offers financing and advising that induces the two partners to provide efficient efforts?*

To answer our question, we consider two cases: in the first case, the project is implemented without debt: the LBO issues all the remaining capital. In the second case, we suppose that the entrepreneur asks for advice from the consultant and for a debt $K - W$ from the bank.

Optimal financial contract without debt
In this paragraph, the LBO fund is assumed to be not wealth-constrained. We suppose that the project is implemented without debt; the LBO fund issue equity $i = K - W$. The agents provide the levels of efforts that enable them to maximize their expected gains which are given by:
$$EU^E = \beta\left[p(e,a)\Delta\theta + \theta^d\right] - u(e) - W \geq 0$$
$$EU^A = (1 - \beta)\left[p(e,a)\Delta\theta + \theta^d\right] - \upsilon(a) - i \geq 0$$

Their reaction functions are given by the first order conditions of EU^E and EU^A. They are written:

$$\beta p_e(e,a)\Delta\theta - u'(e) = 0$$
$$(1 - \beta)p_a(e,a)\Delta\theta - \upsilon'(a) = 0$$

We deduce that the optimal sharing rule $\beta \in \left]0,1\right[$. As in the previous section, the optimal holding contract induces the entrepreneur to maximize the social value of the project under the incentive constraints. Consequently, the entrepreneur's program is given by:

$$\frac{\max V(\beta)}{\beta} = \rho(\beta)\Delta\theta^2 - K - \frac{1}{2}\big[\beta(1-\alpha) + \alpha(1-\beta)\big]\rho(\beta)\Delta\theta^2$$

$$s.t \quad 0\langle\beta\langle1$$

Preposition 3 *Under the condition* $i^c = K - W$ *, the entrepreneur share of benefit is given by:*

$$\beta^c = \begin{cases} \dfrac{\dfrac{(1-\alpha)(2-\alpha) - \sqrt{\alpha(\alpha+1)(1-\alpha)(2-\alpha)}}{2(1-2\alpha)}}{\dfrac{1}{2}} & \text{if } \alpha \in]\ 0,1[/\{\dfrac{1}{2}\} \end{cases}$$

if $\alpha = \dfrac{1}{2}$

This proposition states that when the impact of the LBO fund's effort on the success probability is low, such that $\alpha \in]0,1[$, he perceives a lower share of benefit than the entrepreneur $\left(\beta > \dfrac{1}{2}\right)$. But when this impact is relatively high $\alpha > \dfrac{1}{2}$, his effort is the most efficient. Hence, we must give him the highest share of benefit in order to induce him to provide the highest level of effort.

When $\alpha = \dfrac{1}{2}$, the efforts have the same impact on the success probability, if we fix high share of benefit to one agent, we will induce him to provide an important effort but, at the same time, we will reduce the incentives of the other agent. This is why it is optimal to give them equal shares of benefit such that they are both induced to provide high efforts. Thus, they provide the efforts given by:

$$e^c = \frac{1}{4}\lambda^{\frac{3}{4}}\mu^{\frac{1}{4}}\Delta\theta = \frac{1}{2}e^{FB} \text{ and } a^c = \frac{1}{4}\lambda^{\frac{1}{4}}\mu^{\frac{3}{4}}\Delta\theta = \frac{1}{2}a^{FB}$$

We notice that these efforts are less efficient than the efforts provided when the bank does not participate in the project. This means that the entrepreneur prefers asking for advice and for money from the LBO fund and the bank rather than relying on the LBO fund alone.

The expected gain of the entrepreneur is lower than the optimal social value of the project such that:

$$EU^E = \frac{\alpha\big[\theta(\alpha)\Delta\theta\big]^2}{2\lambda(1-\alpha)} + \theta^d - K\langle V^{FB}$$

When α is high enough $(\alpha \mapsto 1)$, her gain comes close to V^{FB}.

Optimal financial contracts when the LBO fund does not issue equity: the case of a consultant

We suppose now that the LBO fund does not contribute financially into the project but he still spend the effort a. The LBO fund can be considered as a simple consultant. The two partners will ask for the amount $K - W$ from the bank. The participation constraints of the consultant and the bank (PC$_A$) and (PC$_B$) are given by:

$$EU^A = (1-\beta)\big\lfloor p(e,a)(\Delta\theta - D + H) + \theta^d - H\big\rfloor - \upsilon(a) \geq 0$$
$$EU^B = p(e,a)(D - H) + D - (K - W) \geq 0$$

The two constraints are satisfied because of the competition among the consultants and the banks. The optimal efforts are given by the first order conditions of EU^A and EU^E. So, they must be equal to the first best effort As in the previous sections, the entrepreneur maximizes her expected gain under the participation constraints of the bank and the consultant and the incentive constraints.

Preposition 4 *When the bank invests the remaining capital* $I = K - W$, *the entrepreneur and the consultant get equal shares of the benefit:* $\widetilde{\beta} = \dfrac{1}{2}$. *Then, the optimal debt contract is given by:*

- When the project is lowly risky ($\theta^u \langle 2\theta^d)$

$$\widetilde{H} = K - W + \frac{1}{\lambda(1-\alpha)}\phi^2(\alpha)\Delta\theta^2 \text{ and } \widetilde{D} = K - W - \Delta\theta + \frac{1}{\lambda(1-\alpha)}\phi^2(\alpha)\Delta\theta^2$$

- When the project is highly risky $\theta^u \geq 2\theta^d$

$$\widetilde{H} = \theta^d \text{ and } \widetilde{D} = 0$$

This proposition shows that these contracts have the properties of the contracts discussed in propositions (1) and (2): giving the bank decreasing payments induces the entrepreneur and the consultant to exert the first best efforts when θ^u is not very large compared with θ^d. However, this is no longer true when the difference between the project's revenues is very large, to induce them to exert efficient efforts, we must give them high-powered incentives as sharing the revenue of success between them and pledging the entire revenue of failure to the bank. These contracts exhibit the features of a "*live or die*" contract. Despite these incentives, the level of first best efforts are still high, the entrepreneur and the consultant exert second best efforts. In contrast with the proposition (2), whatever the impact of each effort on the success' probability and the quality of the project are, the entrepreneur and the consultant have equal remunerations. We conclude that when the project is not very risky, the identity of the agent providing advice; the LBO fund or the consultant, is irrelevant for the entrepreneur. Hence, the partners spend efficient efforts because the debt is a high powered incentive constraint. The advising and the financing of the LBO fund alone is not sufficient to induce them to provide the first best efforts.

When $\alpha > \dfrac{1}{2}$, we show that the effort exerted by the entrepreneur in the presence of the consultant and the bank is more efficient than her effort when she implements the project with the LBO fund and the bank. In the opposite, the financial investor is better off when he contributes financially into the project, as the LBO fund. The intuition of this result is the following: when the effort of the LBO fund has a significant impact on the probability of success, his share of the benefits is higher than the entrepreneur's share. Hence, the entrepreneur prefers a consultant because they will perceive equal payments. Moreover, we conclude that when the project is lowly risky, the entrepreneur prefers asking for advise from the consultant and for the remaining capital from the bank rather than relying on the LBO fund to advise her and to provide all the money because both agents provide the first best efforts in the first situation. When the project is highly risky, the levels of efforts depend on the impact of the LBO's effort on the performance of the project:

(1) If the efforts have the same impact on the performance of the project, it is optimal to ask separately for advice from a consultant and for money from a bank. Indeed, relying on the LBO fund as a financier and an adviser at the same time does not induce them to exert more efficient efforts. Dealing with one financial investor who offers all the needed tasks is not sufficient to induce them to exert more efficient efforts.

(2) If α is not high enough, without debt, the entrepreneur's effort is more efficient than her effort if the consultant advises her and the bank provides the needed money. However, when her effort does not influence significantly the probability of success, she provides more effort when the project is implemented with the consultant and the bank. In the opposite, the LBO fund provides more efficient effort than the consultant when it contributes significantly on the probability of success.

5. Conclusion

This paper has studied the properties of the optimal financial contracts in LBO. We have focused on the incentive effects of debt in the presence of a double-sided moral hazard problem. The model explains why the entrepreneur prefers asking for money and for advice from the bank and the LBO fund, despite the fact that the latter can advise her and provide funds. We show that all agents have to participate financially in the project and that the payments of the bank are decreasing with the outcome of the project. When the project is not very risky, the entrepreneur and the LBO fund exert the first best efforts and they get equal shares of the outcome of the project. This is no longer true when the acquisition is very risky; the moral hazard problem induces them to provide the second best efforts. They must share the revenue in case of success and pledge the entire revenue to the bank when the project fails.

Exploring in full detail how these optimal contracts are implemented in LBO is an important topic for future research. In such research, it would probably be reasonable to study the use of convertible securities in the presence of a passive financier as the bank. Another topic for further research is how the cash-flow and control rights are allocated when the profitability of the project varies. To our knowledge, research in this direction is still pending. Moreover, while it is optimal to pledge the entire revenue to the bank in case of failure and to let the entrepreneur and the LBO share the revenue, it may incite them to collude: the entrepreneur and the LBO fund may be tempted to inject money into the project.

They aim to mislead the bank: they can tell him that the project has succeeded while it fails. In order to avoid such behavior, Casamatta (2003) assumes that the revenue of the pure financier must be non-decreasing with the outcome of the project. If we consider that the debt payments are decreasing, this makes the question raised in this paper irrelevant since the debt has no incentive impact on efforts.

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