Diversification

Class 10
Financial Management, 15.414

Today

Diversification

- Portfolio risk and diversification
- Optimal portfolios

Reading

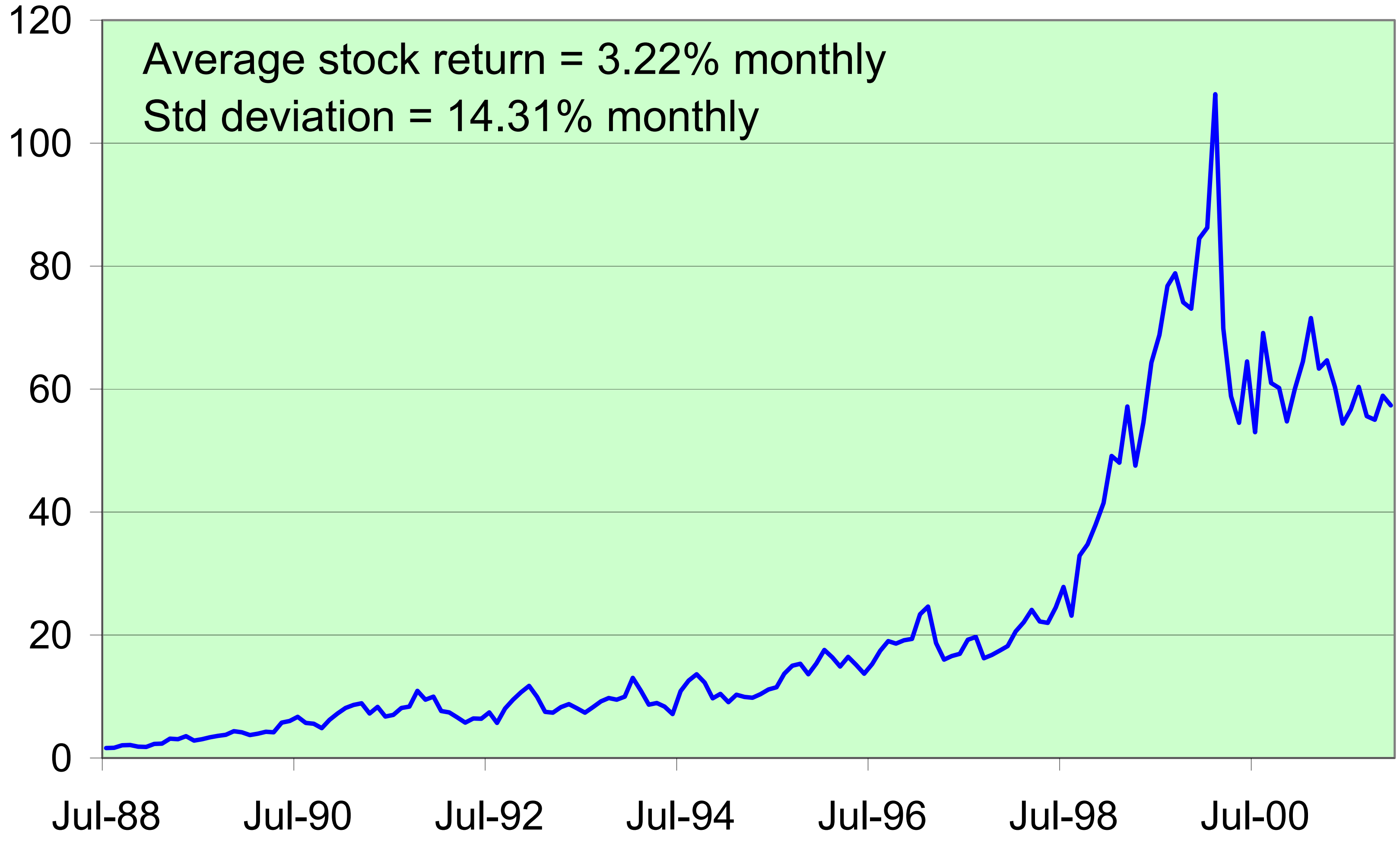
- Brealey and Myers, Chapters 7 and 8.1

Example

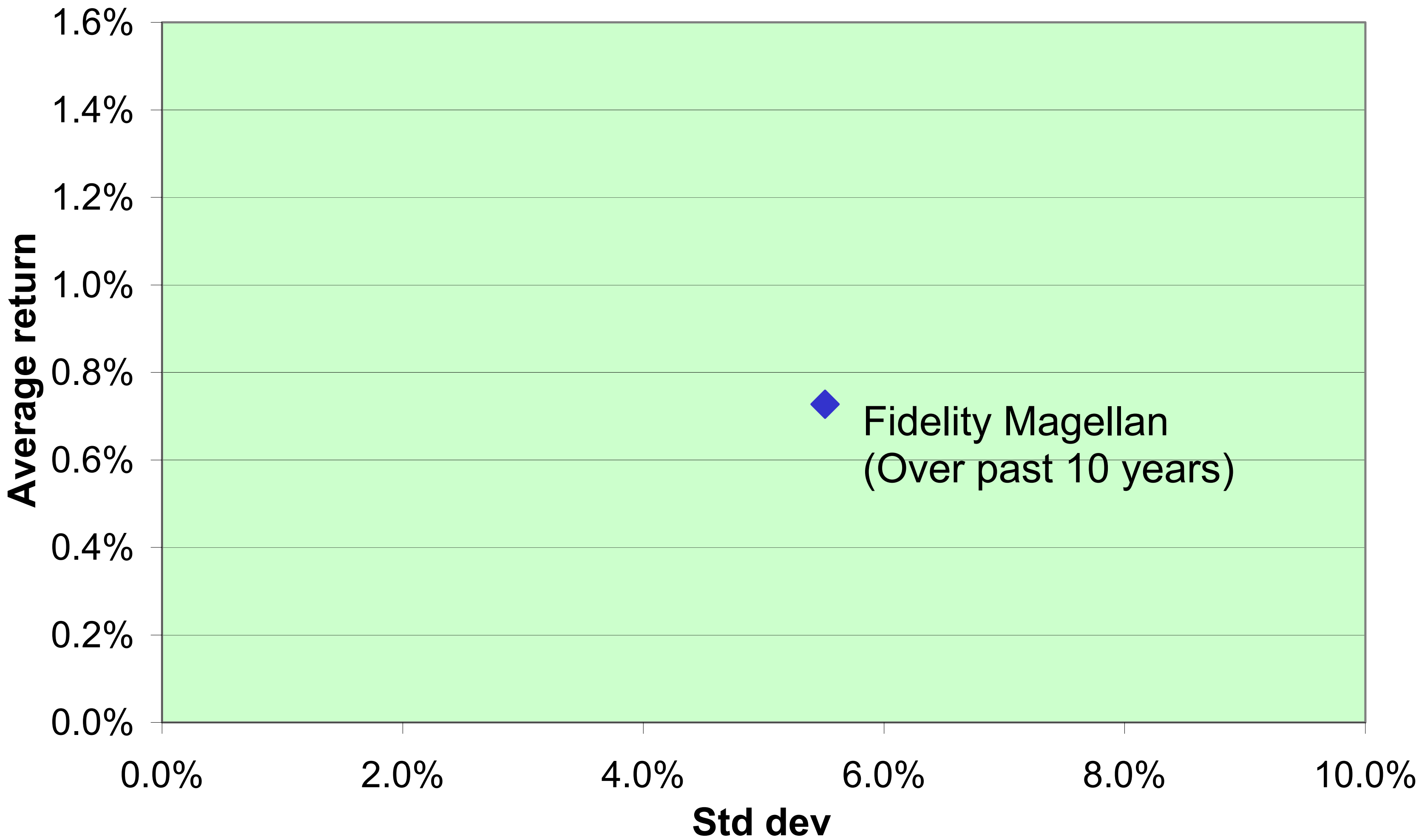
Fidelity Magellan, a large U.S. stock mutual fund, is considering an investment in Biogen. Biogen has been successful in the past, but the payoffs from its current R&D program are quite uncertain. How should Magellan's portfolio managers evaluate the risks of investing in Biogen?

Magellan can also invest Microsoft. Which stock is riskier, Microsoft or Biogen?

Biogen stock price, 1988 – 2001



Fidelity Magellan



Example

Exxon is bidding for a new oil field in Canada. Exxon's scientist estimate that there is a 40% chance the field contains 200 million barrels of extractable oil and a 60% chance it contains 400 million barrels.

The price of oil is $30 and Exxon would have to spend $10 / barrel to extract the oil. The project would last 8 years.

What are the risks associated with this project? How should each affect the required return?

Plan

Portfolio mean and variance

- Two stocks*
- Many stocks*

How much does a stock contribute to the portfolio's risk?

How much does a stock contribute to the portfolio's return?

What is the best portfolio?

* Same analysis applies to portfolios of projects

Portfolios

Two stocks, A and B

You hold a portfolio of A and B. The fraction of the portfolio invested in A is w_A and the fraction invested in B is w_B.

Portfolio return = $R_P = w_A R_A + w_B R_B$

What is the portfolio's expected return and variance?

Portfolio

$$E[R_P] = w_A E[R_A] + w_B E[R_B]$$

$$\text{var}(R_P) = w_A^2 \,\text{var}(R_A) + w_B^2 \,\text{var}(R_B) + 2\, w_A w_B \,\text{cov}(R_A, R_B)$$

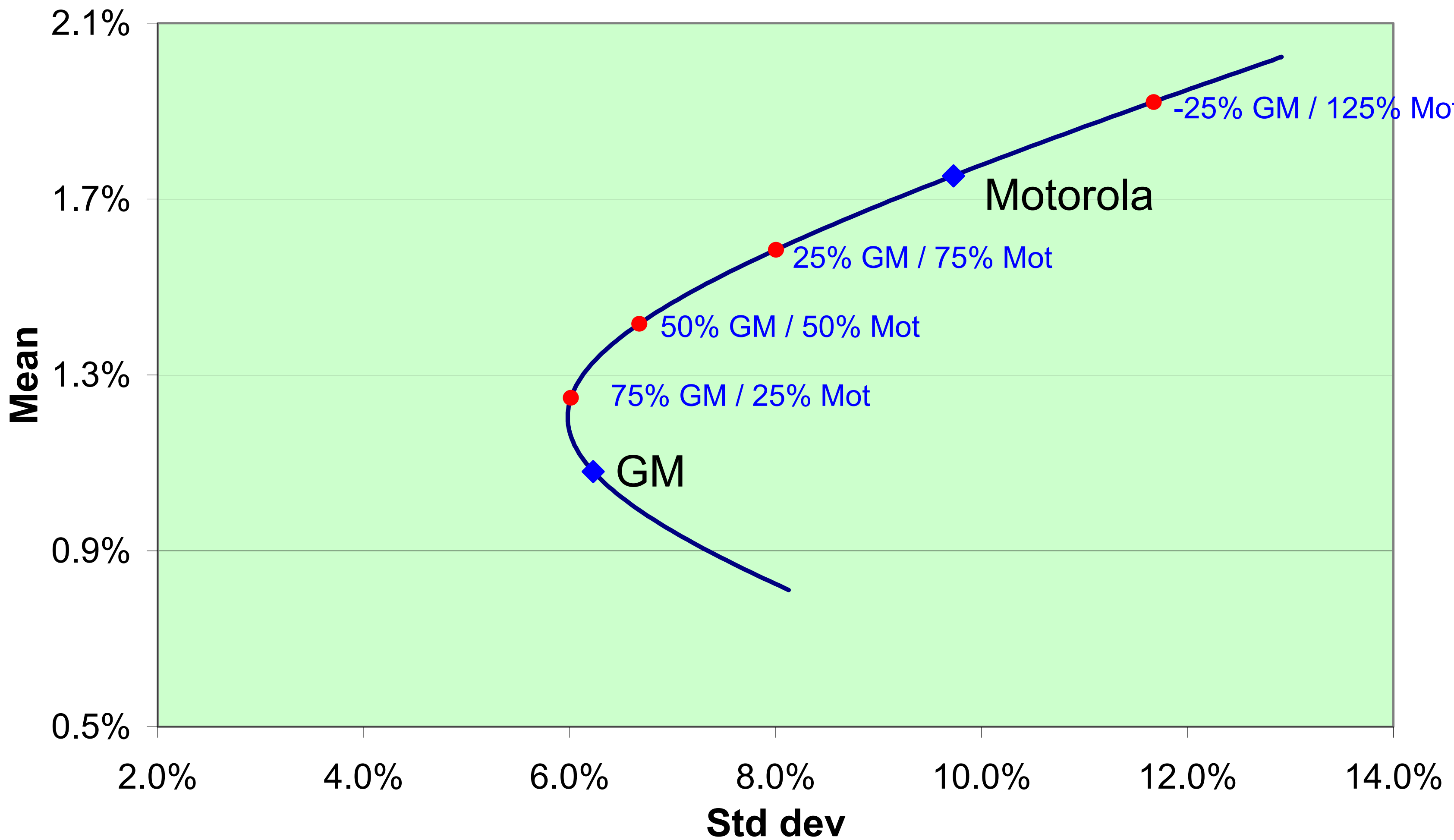
Example 1

Over the past 50 years, Motorola has had an average monthly return of 1.75% and a std. dev. of 9.73%. GM has had an average return of 1.08% and a std. dev. of 6.23%. Their correlation is 0.37. How would a portfolio of the two stocks perform?

- **$E[R_P]$** $= w_{GM}\, 1.08 + w_{Mot}\, 1.75$
- **$var(R_P)$** $= w_{GM}^2\, 6.23^2 + w_{Mot}^2\, 9.73^2 + 2\, w_{Mot}\, w_{GM}\, (0.37 \times 6.23 \times 9.73)$

w_{Mot}	w_{GM}	$E[R_P]$	$var(R_P)$	$stdev(R_P)$
0	**1**	1.08	38.8	6.23
0.25	**0.75**	1.25	36.2	6.01
0.50	**0.50**	1.42	44.6	6.68
0.75	**0.25**	1.58	64.1	8.00
1	**0**	1.75	94.6	9.73
1.25	**-0.25**	1.92	136.3	11.67

GM and Motorola



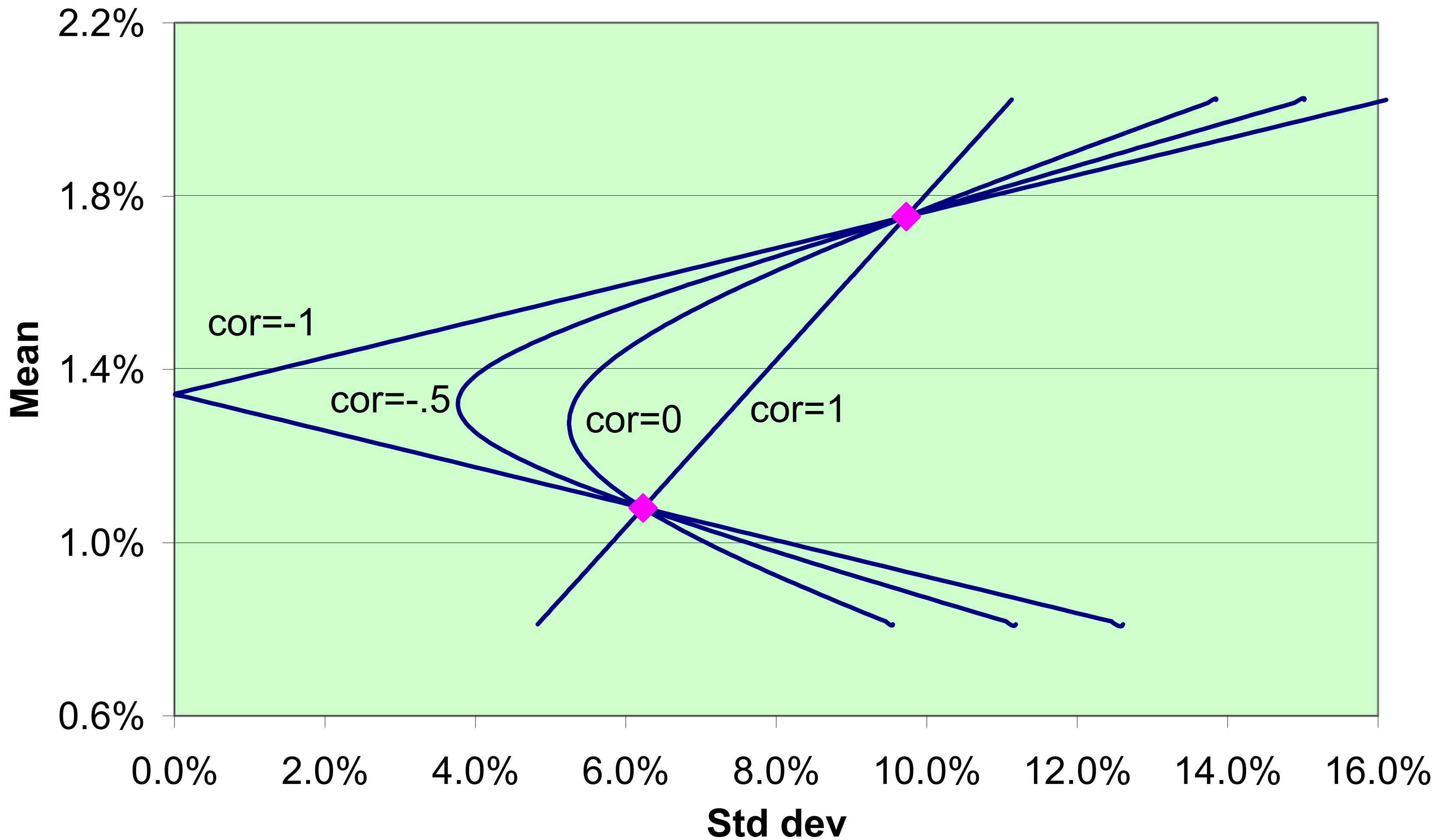
Example 1, cont.

Suppose the correlation between GM and Motorola changes. What if it equals –1.0? 0.0? 1.0?

- **$E[R_P]$** $= w_{GM}\,1.08 + w_{Mot}\,1.75$
- **$var(R_P)$** $= w_{GM}^2\,6.23^2 + w_{Mot}^2\,9.73^2 + 2\,w_{Mot}\,w_{GM}\,(corr \times 6.23 \times 9.73)$

			Std dev of portfolio		
w_{Mot}	**w_{GM}**	$E[R_P]$	corr = -1	corr = 0	corr = 1
0	**1**	1.08%	6.23%	6.23%	6.23%
0.25	**0.75**	1.25	2.24	5.27	7.10
0.50	**0.50**	1.42	1.75	5.78	7.98
0.75	**0.25**	1.58	5.74	7.46	8.85
1	**0**	1.75	9.73	9.73	9.73

GM and Motorola: Hypothetical correlations



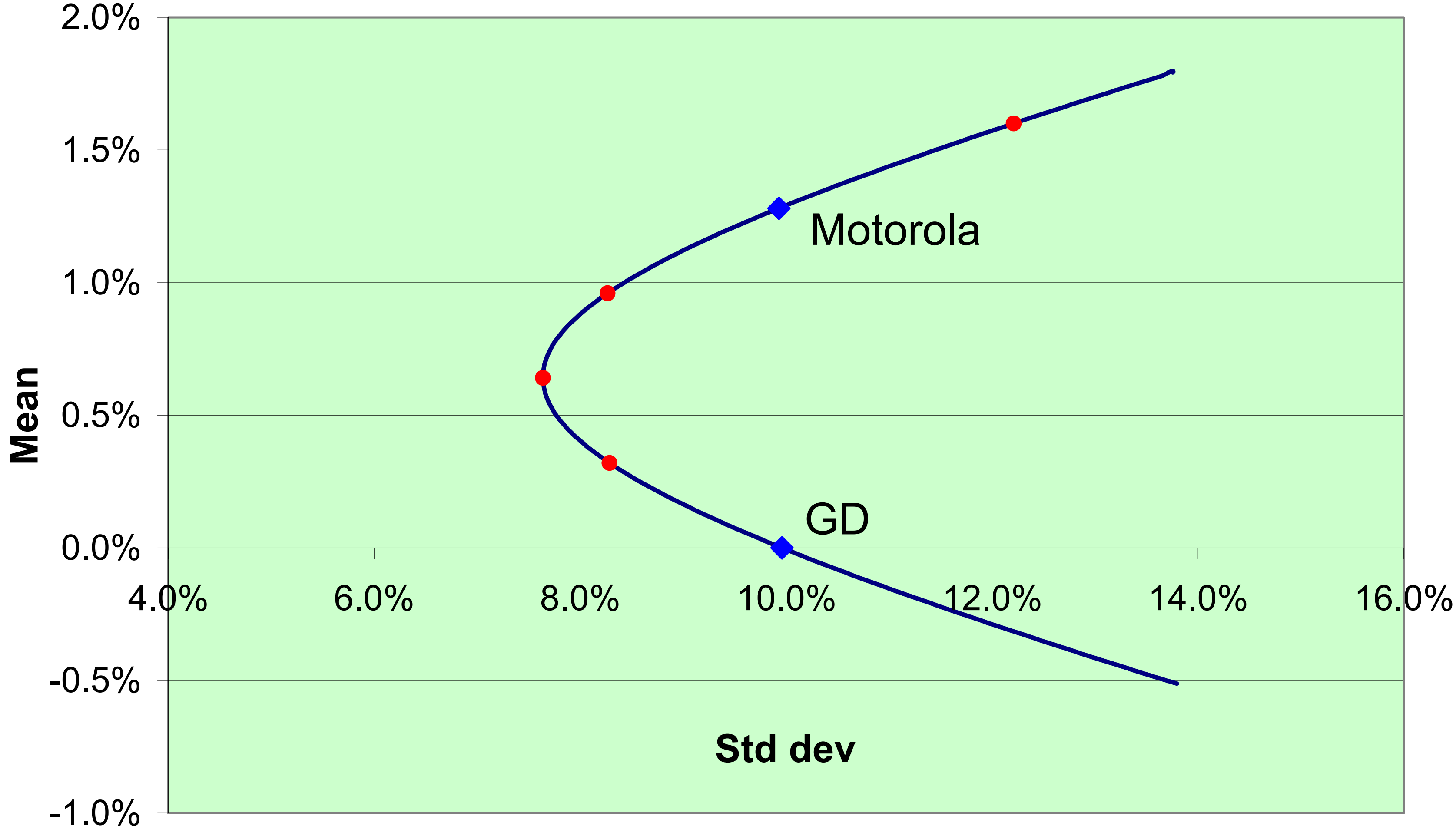
Example 2

In 1980, you were thinking about investing in GD. Over the subsequent 10 years, GD had an average monthly return of 0.00% and a std dev of 9.96%. Motorola had an average return of 1.28% and a std dev of 9.33%. Their correlation is 0.28. How would a portfolio of the two stocks perform?

- $\mathbf{E[R_P]} = w_{GD}\,0.00 + w_{Mot}\,1.28$
- $\mathbf{var(R_P)} = w_{GD}^2\,9.96^2 + w_{Mot}^2\,9.33^2 + 2\,w_{Mot}\,w_{GD}\,(0.28\times9.96\times9.33)$

w_{Mot}	w_{GD}	$E[R_P]$	$var(R_P)$	$stdev(R_P)$
0	1	0.00	99.20	9.96
0.25	0.75	0.32	71.00	8.43
0.50	0.50	0.64	59.57	7.72
0.75	0.25	0.96	64.92	8.06
1	0	1.28	87.05	9.33

GD and Motorola



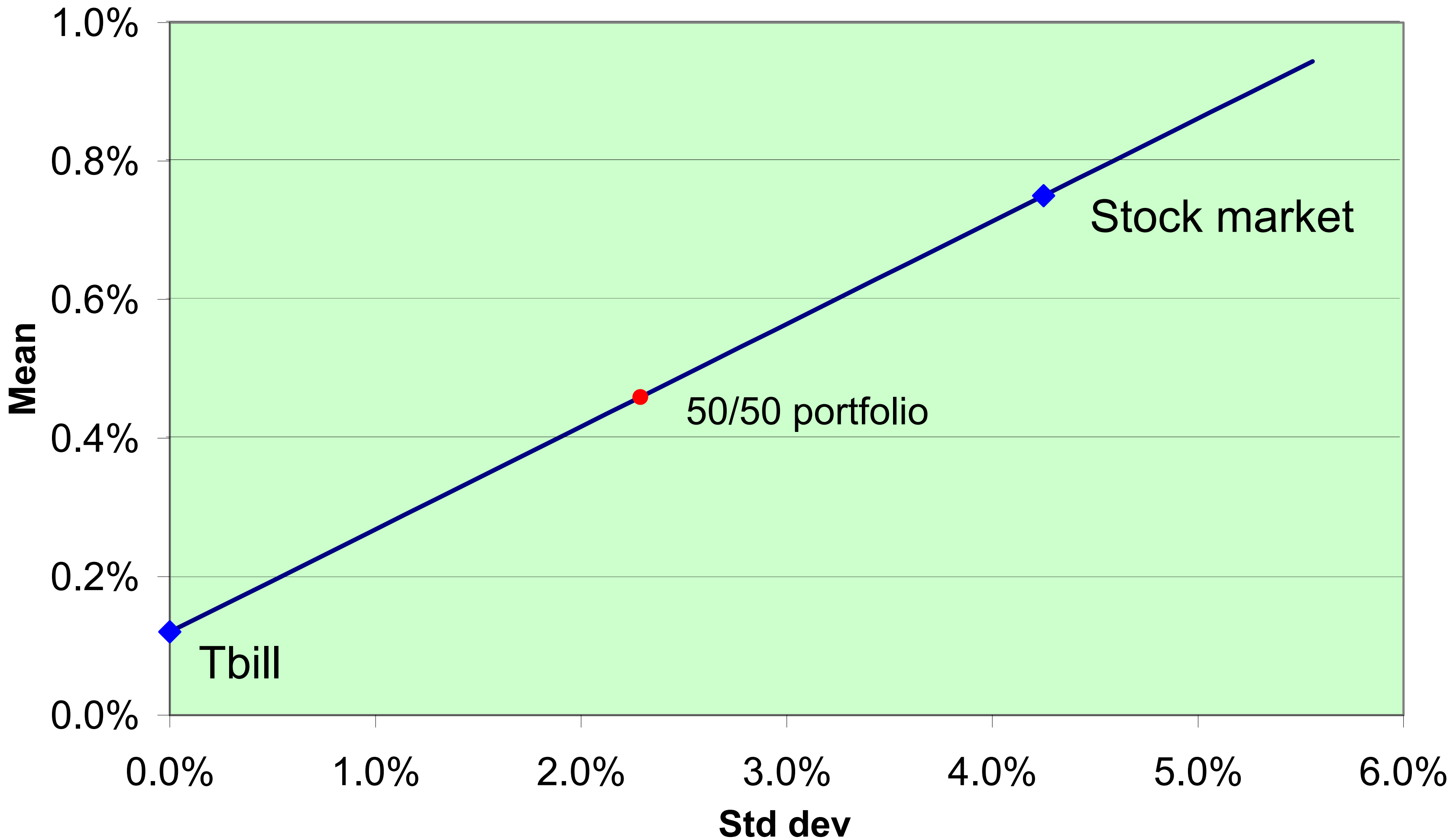
Example 3

You are trying to decide how to allocate your retirement savings between Treasury bills and the stock market. The Tbill rate is 0.12% monthly. You expect the stock market to have a monthly return of 0.75% with a standard deviation of 4.25%.

- $\mathbf{E[R_P]} = w_{Tbill}\, 0.12 + w_{Stk}\, 0.75$
- $\mathbf{var(R_P)} = \underbrace{w_{Tbill}^2\, 0.0^2 + w_{Stk}^2\, 4.25^2 + 2\, w_{Tbill}\, w_{stk}\, (0.0 \times 0.0 \times 4.25)}_{w_{Stk}^2\, 4.25^2}$

w_{Stk}	w_{Tbill}	$E[R_P]$	$var(R_P)$	$stdev(R_P)$
0	**1**	0.12	0.00	0.00
0.33	**0.67**	0.33	1.97	1.40
0.67	**0.33**	0.54	8.11	2.85
1	**0**	0.75	18.06	4.25

Stocks and Tbills



Many assets

Many stocks, R_1, R_2, …, R_N

You hold a portfolio of stocks 1, …, N. The fraction of your wealth invested in stock 1 is w_1, invested in stock 2 is w_2, etc.

Portfolio return = $R_P = w_1 R_1 + w_2 R_2 + \ldots + w_N R_N = \sum_i w_i R_i$

Portfolio mean and variance

$E[R_P] = \sum_i w_i E[R_i]$ (weighted average)

$\text{var}(R_P) = \sum_i w_i^2 \text{var}(R_i) + \sum \sum_{i \neq j} w_i w_j \text{cov}(R_i, R_j)$

Many assets

Variance = sum of the matrix

	Stk 1	Stk 2	$\cdots$	Stk N
Stk 1	$w_1^2 var(R_1)$	$w_1 w_2 cov(R_1, R_2)$	$\cdots$	$w_1 w_N cov(R_1, R_N)$
Stk 2	$w_1 w_2 cov(R_1, R_2)$	$w_2^2 var(R_2)$	$\cdots$	$w_2 w_N cov(R_2, R_N)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Stk N	$w_1 w_N cov(R_1, R_N)$	$w_2 w_N cov(R_2, R_N)$	$\cdots$	$w_N^2 var(R_N)$

The matrix contains N^2 terms

- N are variances
- N(N-1) are covariances

In a diversified portfolio, covariances are more important than variances. A stock's variance is less important than its covariance with other stocks.

Fact 1: Diversification

Suppose you hold an equal-weighted portfolio of many stocks (inves-ting the same amount in every stock). What is the variance of your portfolio?

- Portfolio of N assets, $w_i = 1/N$
- $\text{var}(R_P) = \frac{1}{N}\text{Avg. variance} + \frac{N-1}{N}\text{Avg. covariance}$

For a diversified portfolio, variance is determined by the average covariance among stocks.

An investor should care only about common variation in returns ('systematic' risk). Stock-specific risk gets diversified away.

Example

The average stock has a monthly standard deviation of 10% and the average correlation between stocks is 0.40. If you invest the same amount in each stock, what is variance of the portfolio? What if the correlation is 0.0? 1.0?

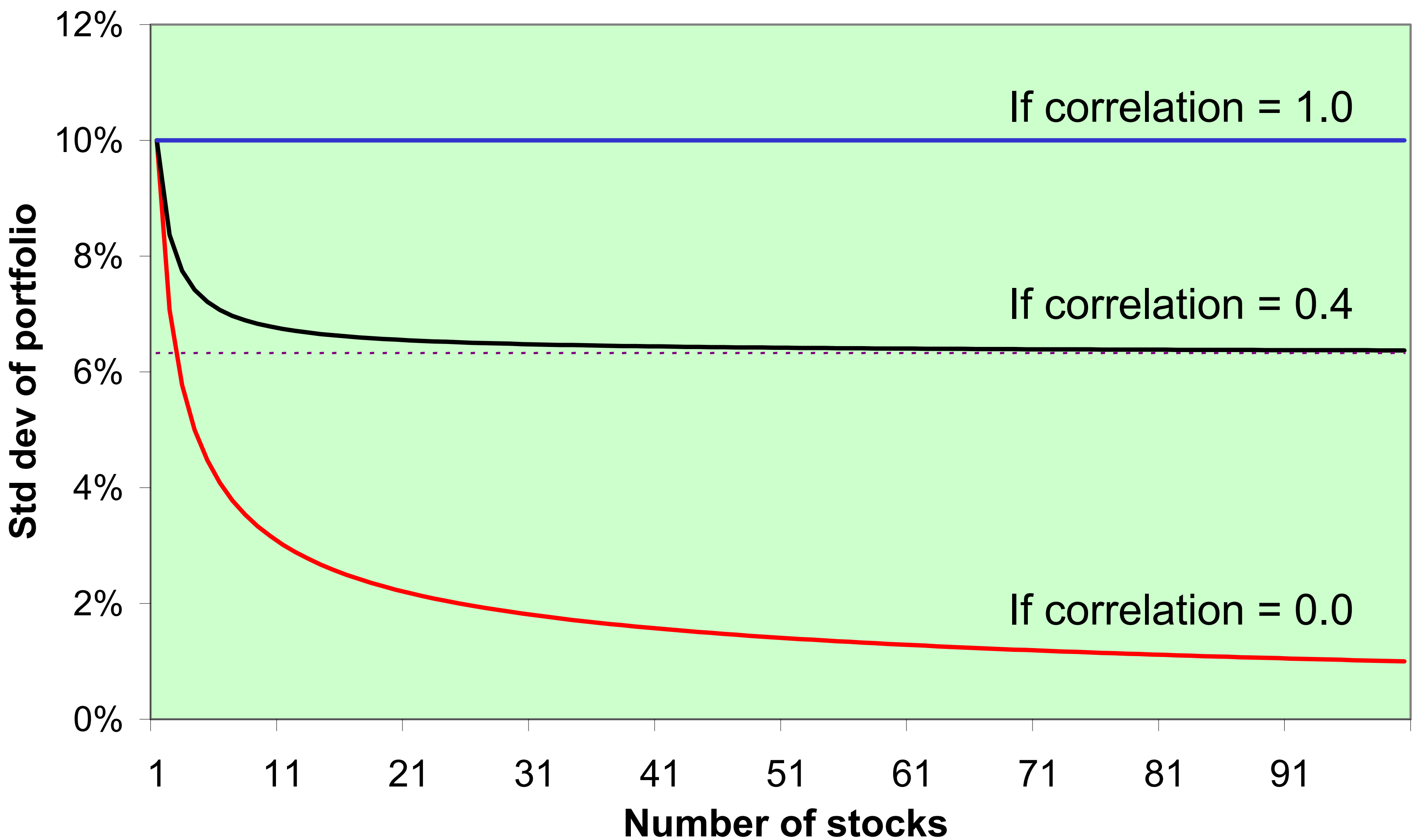
- **$\mathrm{cov}(R_i, R_j)$** $=$ correlation $\times$ stdev(R_i) $\times$ stdev(R_j)

  $= 0.40 \times 0.10 \times 0.10 = 0.004$

- **$\mathrm{var}(R_P)$** $= \dfrac{1}{N} 0.10^2 + \dfrac{N-1}{N} 0.004 \quad \Rightarrow \quad 0.004$ if N is large

- **$\mathrm{stdev}(R_P)$** $\approx \sqrt{0.004} = 6.3\%$

Diversification



Fact 2: Efficient portfolios

With many assets, any portfolio inside a bullet-shaped region is feasible.

- The **minimum-variance** boundary is the set of portfolios that minimize risk for a given level of expected returns.*

- The **efficient frontier** is the top half of the minimum-variance boundary.

* On a graph, the minimum-variance boundary is an hyperbola.

Example

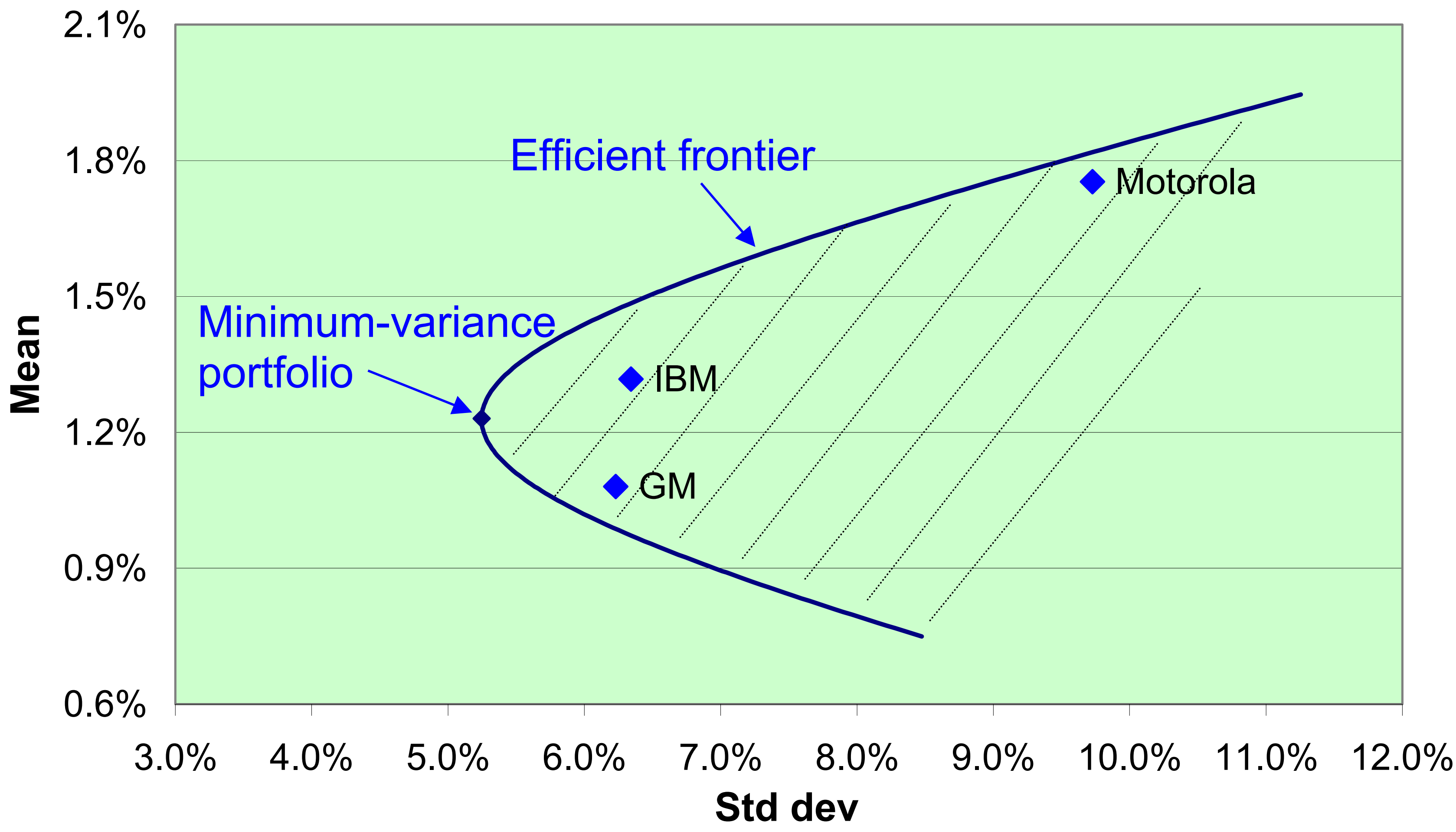
You can invest in any combination of GM, IBM, and Motorola. Given the following information, what portfolio would you choose?

			Variance / covariance		
Stock	Mean	Std dev	GM	IBM	Motorola
GM	1.08	6.23	38.80	16.13	22.43
IBM	1.32	6.34	16.13	40.21	23.99
Motorola	1.75	9.73	22.43	23.99	94.63

$$\mathbf{E[R_P]} = (w_{GM} \times 1.08) + (w_{IBM} \times 1.32) + (w_{Mot} \times 1.75)$$

$$\mathbf{var(R_P)} = (w_{GM}^2 \times 6.23^2) + (w_{IBM} \times 6.34^2) + (w_{Mot}^2 \times 9.73^2) +$$
$$(2 \times w_{GM} \times w_{IBM} \times 16.13) + (2 \times w_{GM} \times w_{Mot} \times 22.43) +$$
$$(2 \times w_{IBM} \times w_{Mot} \times 23.99)$$

Feasible portfolios



Fact 3

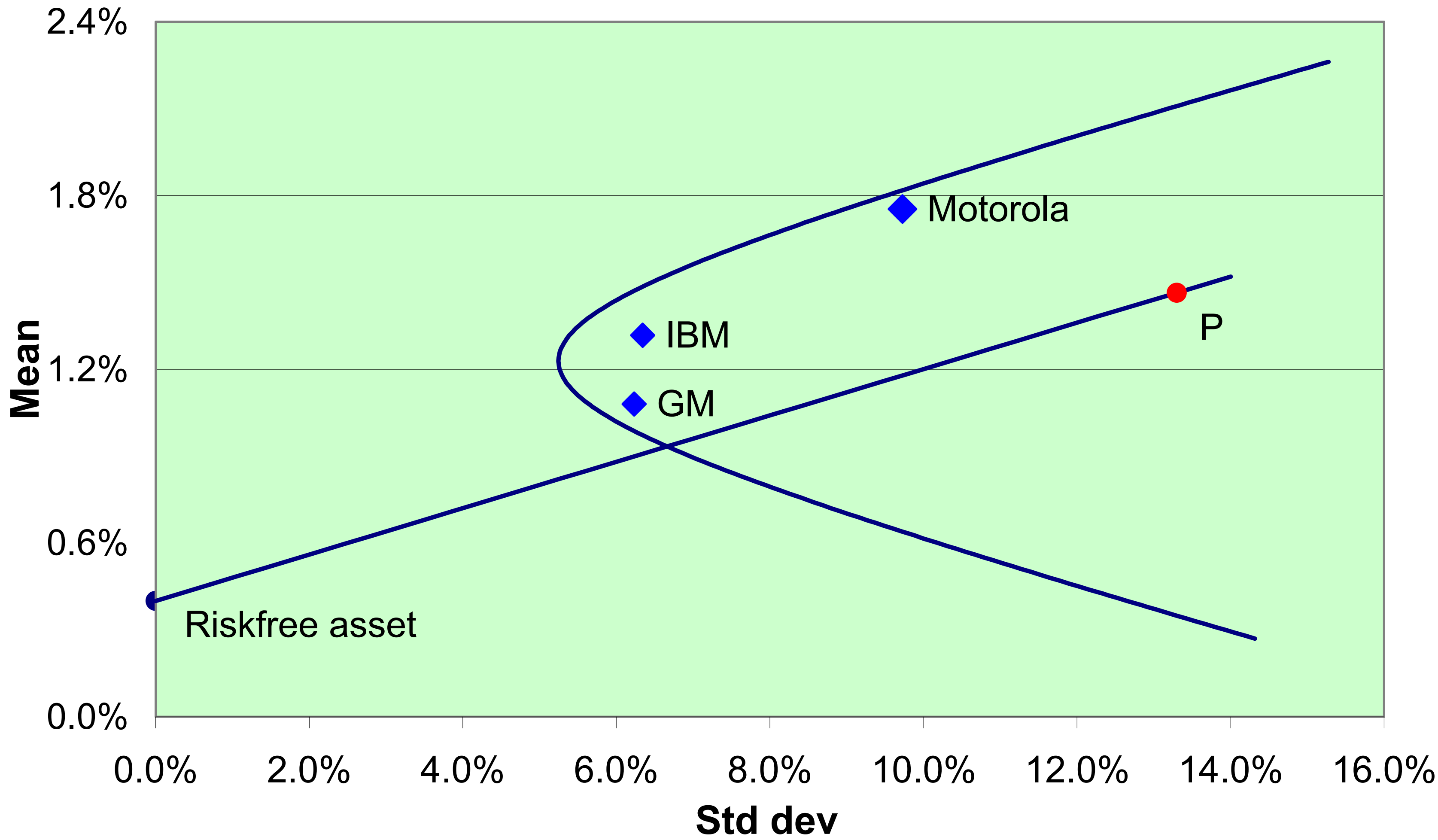
Tangency portfolio

If there is also a riskless asset (Tbills), all investors should hold exactly the same stock portfolio!

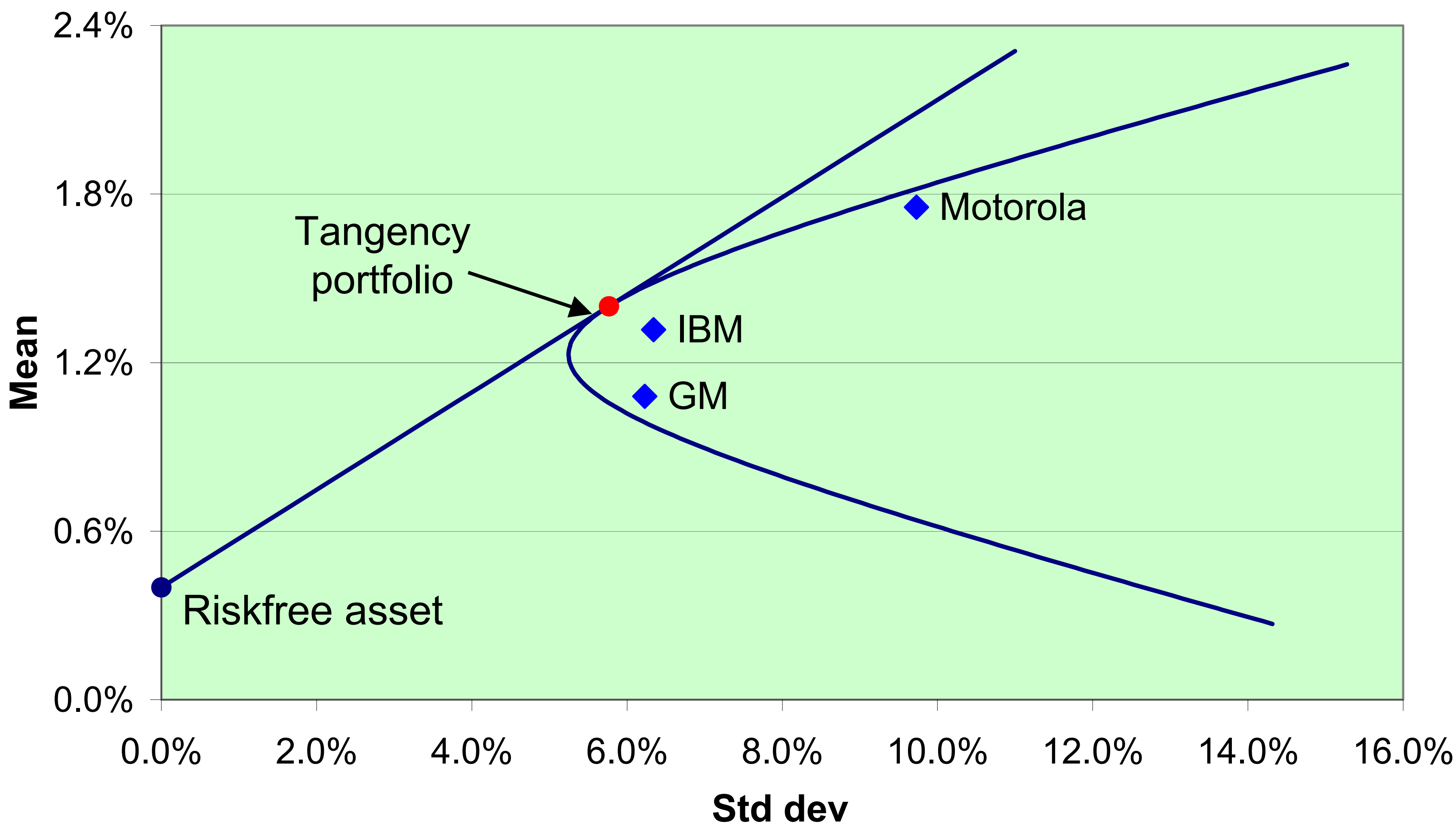
All efficient portfolios are combinations of the riskless asset and a unique portfolio of stocks, called the tangengy portfolio.*

* Harry Markowitz, Nobel Laureate

Combinations of risky and riskless assets



Optimal portfolios with a riskfree asset



Fact 3, cont.

With a riskless asset, the optimal portfolio maximizes the slope of the line.

The tangency portfolio has the maximum Sharpe ratio of any portfolio, where the Sharpe ratio is defined as

$$\textbf{Sharpe ratio} = \frac{E[R_P] - r_f}{\sigma_P}$$

Put differently, the tangency portfolio has the best risk-return trade-off of any portfolio.

Aside
'Alpha' is a measure of a mutual fund's risk-adjusted performance. A mutual fund should hold the tangency portfolio if it wants to maximize its alpha.

Summary

- **Diversification reduces risk.** The standard deviation of a portfolio is always less than the average standard deviation of the individual stocks in the portfolio.
- **In diversified portfolios, covariances among stocks are more important than individual variances.** Only systematic risk matters.
- **Investors should try to hold portfolios on the efficient frontier.** These portfolios maximize expected return for a given level of risk.
- **With a riskless asset, all investors should hold the tangency portfolio.** This portfolio maximizes the trade-off between risk and expected return.